M. Math/ B3 Ist Semester 2009-10 Final Exam Topology

Answer all the questions. Each question is worth 6 points. You may state correctly and use any result proved in the class. However if an answer is an almost immediate consequence of the stated result, such a result also need to be proved.

All topological spaces $(X, \mathcal{T})$ are assumed to be Hausdorff.

1) Let $\{x_\alpha\}_{\alpha \in \Delta} \subset X$ be a net. Let $\mathcal{A}$ be a family of subsets of X such that the intersection of any two members of $\mathcal{A}$ contains a member of $\mathcal{A}$. Suppose the net is frequently in each member of $\mathcal{A}$. Show that there is a subnet of $\{x_\alpha\}_{\alpha \in \Delta}$ which is eventually in each member of $\mathcal{A}$.

2) Give an example of a separable topological space that is not metrizable.

3) Show that the set of rational numbers is not a G_δ set in the set of real numbers with the usual topology.

4) Let (X, d) be a metric space such that every countable collection of closed sets, any finitely many of them intersect, has non-empty intersection. Show that X is compact.

5) Show that the union of any finitely many connected subsets of X having at least one point in common is also a connected set.

6) Show that every subspace of a regular space is regular.

7) Let $(X, \mathcal{T})$ be a regular space. Suppose X has a basis $\mathcal{B}$ such that $\mathcal{B} = \bigcup \mathcal{B}_n$, where each $\mathcal{B}_n$ is a neighborhood finite family. Show that every open set is an F_σ set.

8) Let $(X, \mathcal{T})$ be a locally compact non-compact space. Give complete details to show that $C_0(X)$ is a complete metric space.

9) Compute the fundamental group of the Möbius strip.

10) Let (X, x) and (Y, y) be two pointed topological spaces. Show

$$\pi_1(X \times Y, (x, y)) \cong \pi_1(X, x) \times \pi_1(Y, y).$$

Use this to compute the fundamental group of $R^2 - \{(0, 0)\}$.